Assginment 1 Submission deadline: 6th Mar 2025, 11:30 pm

1. State whether each of the following observations is an example of discrete or continuous data:

   1.1) **(2½ points)** The number of patients in a hospital.

   1.1) _____

   1.2) **(2½ points)** The concentration of NaCl in a sample of water.

   1.2) _____

   1.3) **(2½ points)** The number of mRNA molecules in a cell.

   1.3) _____

   1.4) **(2½ points)** The number of pages in a textbook.

   1.4) _____

2. **Canadian cigarettes**: The declared concentrations of tar and nicotine for 35 brands of Canadian cigarettes are shown below. The values represent milligrams per cigarettes. For your convenience, you can download the source file of the table by clicking here.

Brand	tar	nicotine	Brand	tar	nicotine	Brand	tar	nicotine
Brand1	3	0.3	Brand13	8	0.9	Brand25	16	1.3
Brand2	14	1.2	Brand14	12	1.3	Brand26	11	1.0
Brand3	16	1.3	Brand15	13	1.2	Brand27	16	1.3
Brand4	10	1.0	Brand16	15	1.3	Brand28	16	1.3
Brand5	18	1.4	Brand17	12.9	0.89	Brand29	19	1.4
Brand6	13	1.2	Brand18	16	1.2	Brand30	10	1.1
Brand7	1	0.1	Brand19	17	1.3	Brand31	16	1.2
Brand8	10	0.9	Brand20	4	0.4	Brand32	16	1.2
Brand9	16	1.3	Brand21	13	1.1	Brand33	9	1.0
Brand10	13	1.1	Brand22	3	0.4	Brand34	4	0.5
Brand11	15	1.2	Brand23	1	0.2	Brand35	0.7	0.09
Brand12	12	1.0	Brand24	13	1.1			

   2.1) **(2½ points)** Produce a box plot of the declared concentrations of tar per cigarette.

2.2) **(2½ points)** Describe the distribution of values.

2.3) **(2½ points)** Construct a scatter plot of the concentration of tar versus the concentration of nicotine. Label the axes appropriately.

2.4) **(2½ points)** Does there appear to be relationship between these two quantities?

3. **(4 points)** **The relation of two events:** Li Lei has n $(n \geqslant 1)$ coins, and Han Meimei has $n+1$ coins. They flip all of their own coins at the same time, observe their own results and make comparisons in the end. Let events $\boldsymbol{A}$ = { Han Meimei has more **H**-s than Li Lei } and $\boldsymbol{B}$ = { Han Meimei has more **T**-s than Li Lei }. Are events $\boldsymbol{A}$ and $\boldsymbol{B}$ mutually exclusive? Are they collectively exhaustive?

4. **Rolling an unusual die**: Li Lei has a peculiar pair of four-sided dice. One is in red colour, and the other is in blue colour. When he rolls the dice, the probability of any particular outcome is proportional to the sum of the results of each die. All outcomes that result in a particular sum are equally likely.

   4.1) **(5 points)** What is the sample space for rolling the dice?

   4.2) **(7½ points)** Let event $\boldsymbol{A} = \{$ the sum of the two dice is 8 $\}$, what is $\mathbb{P}(A)$?

   4.2) _____

   4.3) **(7½ points)** Let event $\boldsymbol{B} = \{$ the sum of the two dice is odd $\}$, what is $\mathbb{P}(B)$?

   4.3) _____

5. **(10 points)** **Dating**: Han Meimei and Li Lei have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?

6. **A Game About Letters**: Suppose Han Meimei chooses a letter at random from the word "SUSTECH" and then randomly chooses a letter from the word "SCIENCE".

   6.1) **(3 points)** Write the sample space of this experiment.

   6.2) **(3 points)** What is the probability of getting the outcome "HE"?

   6.3) **(4 points)** Let event $\boldsymbol{A}$ = { At least one letter is S } and compute $\mathbb{P}(A)$.

7. **Patient Codes**: A hospital administrator codes incoming patients suffering fever according to whether they have medical insurance (coding 1 if they do and 0 if they do not) and according to their body temperature, which is rated as L ($37-38$ °C), M

(38 − 39 °C), H (39 − 40 °C) and U (40 + °C). For example, a patient code of "**1M**" indicates the patient has medical insurance and has a fever of 38 − 39 °C. Consider an experiment that consists of the coding of a random patient.

7.1) **(2 points)** Write the sample space of this experiment.

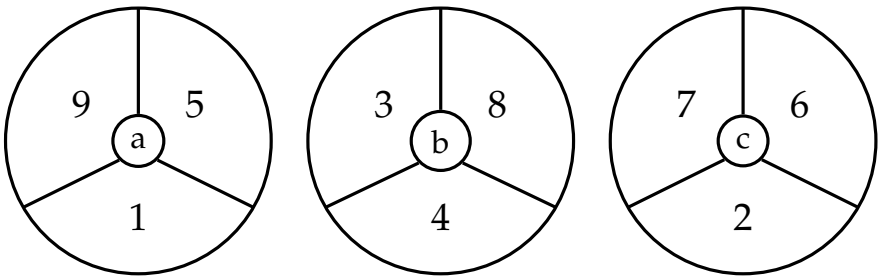
7.2) **(2 points)** Let event $\boldsymbol{A}$ = { The patient's body temperature is greater than 39 °C }. Write all the outcomes in $\boldsymbol{A}$.

7.3) **(2 points)** Let event $\boldsymbol{B}$ = { The patient does not have medical insurance }. Write all the outcomes in $\boldsymbol{B}$.

7.4) **(2 points)** Write all the outcomes in $\boldsymbol{B}^C \cup \boldsymbol{A}$.

7.5) **(2 points)** If you want to calculate $\mathbb{P}(A)$ or $\mathbb{P}(B)$, can you use the discrete uniform law? Why or why not?

8. **Transitive & Non-transitive properties**: When we talk about real numbers, we have transitive properties. For example, x, y, z are real numbers and if $x = y$ and $y = z$, then we know that $x = z$. Similarly, if $x > y$ and $y > z$, then we know that $x > z$. However, when it comes to the **"Rock-Paper-Scissors"** game, we have non-transitive properties: rock beats scissors, scissors beat paper and paper beats rock. Now consider a new game involved in two players: Li Lei and Han Meimei. Li Lei chooses one of the three spinners pictured below, then Han Meimei chooses one of the remaining two spinners. Then they both spin their spinner, and the one that lands on the higher number is declared the winner. Assuming that each spinner is equally likely to land in any of its 3 regions.



8.1) **(4 points)** Does this game have a transitive or non-transitive property?

8.2) **(3 points)** If Li Lei chooses spinner ⓐ, what should Han Meimei choose to have a better chance of winning?

9. **Archery**: Let's revisit the archery example in the lecture. The target board is shown at the end of the document. It consists of 10 co-centred circles. The radius of the inner-most circle is 0.1, and the radius of the outer-most circle is 1. The circles are equally

spaced, i.e. the distance between any two consecutive circles is 0.1. The score you will get when your arrow hits any particular ring is indicated below. Assume you are quite skilled, and you will always shoot on target. In addition, you can place the arrow uniformly on the target board (i.e., the probability of the arrow falling in a given region is proportional to its area). Answer the following questions:

9.1) **(3 points)** What is the probability of getting a score of 10 when you randomly shoot once?

9.2) **(3 points)** When we look at the position of the arrow, what is the sample space? Is it countable or uncountable?

9.3) **(3 points)** When we look at the score you get, what is the sample space? Is it countable or uncountable?

9.4) **(5 points)** What is the average score you will get if you shoot the target board randomly for a large number of times?

9.5) **(5 points)** You friend is twice more likely to shoot in the right half of the target board than in the left half. Across each half, the arrow falls uniformly in that region. Answer questions 9.1 and 9.2 for your friend's shooting.