Lecture 7 More On The Bayes' Theorem

BIO210 Biostatistics

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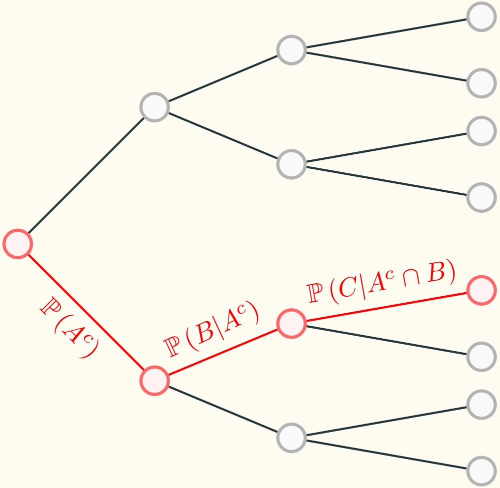
SUSTech · SCHOOL OF
LIFE SCIENCES

The Multiplication Rule

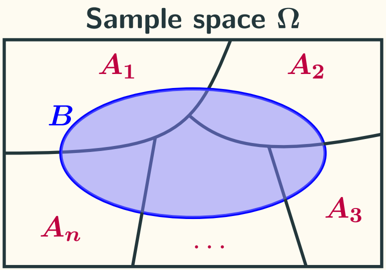
$$\mathbb{P}\left(\cap_{i=1}^{n} A_i\right) = \mathbb{P}\left(A_1\right) \cdot$$
$$\mathbb{P}\left(A_2 | A_1\right) \cdot$$
$$\mathbb{P}\left(A_3 | A_1 \cap A_2\right) \cdot$$
$$\mathbb{P}\left(A_4 | A_1 \cap A_2 \cap A_3\right) \cdot$$
$$\ldots$$
$$\mathbb{P}\left(A_n | \cap_{i=1}^{n-1} A_i\right)$$

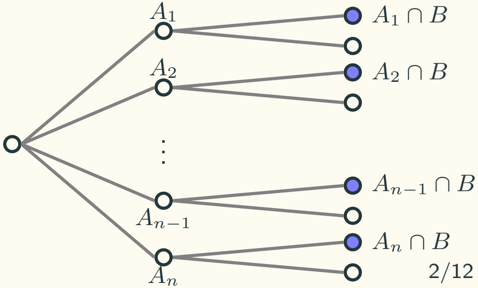


Conditional Probability

The Total Probability Rule

$$
\begin{aligned}
\mathbb{P}(B) &= \mathbb{P}[(A_1 \cap B) \cup (A_2 \cap B) \cup \ldots \cup (A_n \cap B)] \\
&= \mathbb{P}(A_1 \cap B) + \mathbb{P}(A_2 \cap B) + \ldots + \mathbb{P}(A_n \cap B) \\
&= \sum_{i=1}^{n} \mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)
\end{aligned}
$$



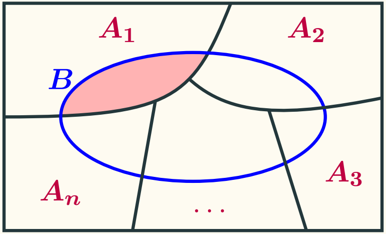


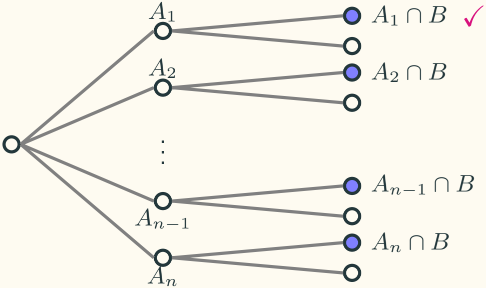
Bayes' Theorem

$$\mathbb{P}(A_i|B) = \frac{\mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}{\mathbb{P}(B)}$$

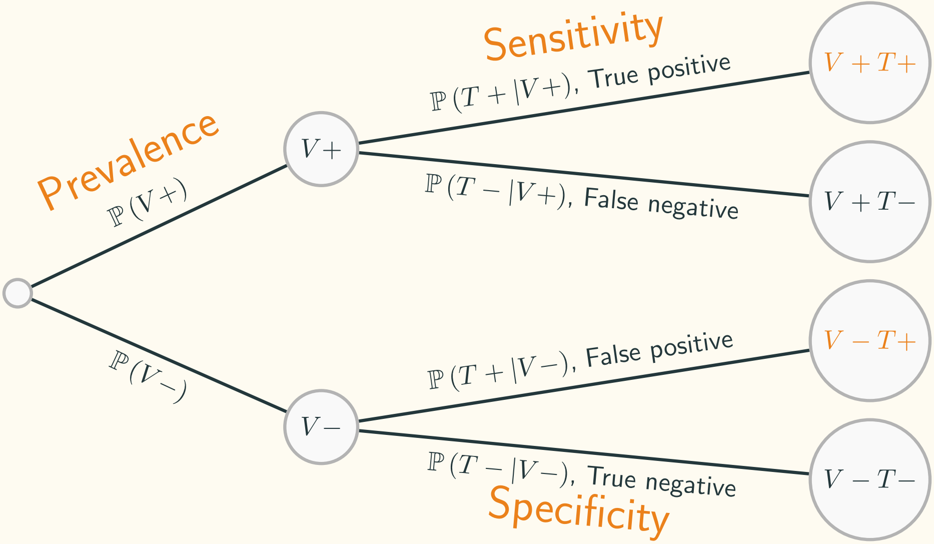
$$= \frac{\mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}{\sum_{i=1}^{n} \mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}$$

Sample space Ω





Virus Detection



Who Is Steve

ARTICLES

Judgment under Uncertainty: Heuristics and Biases

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- Hide authors and affiliations

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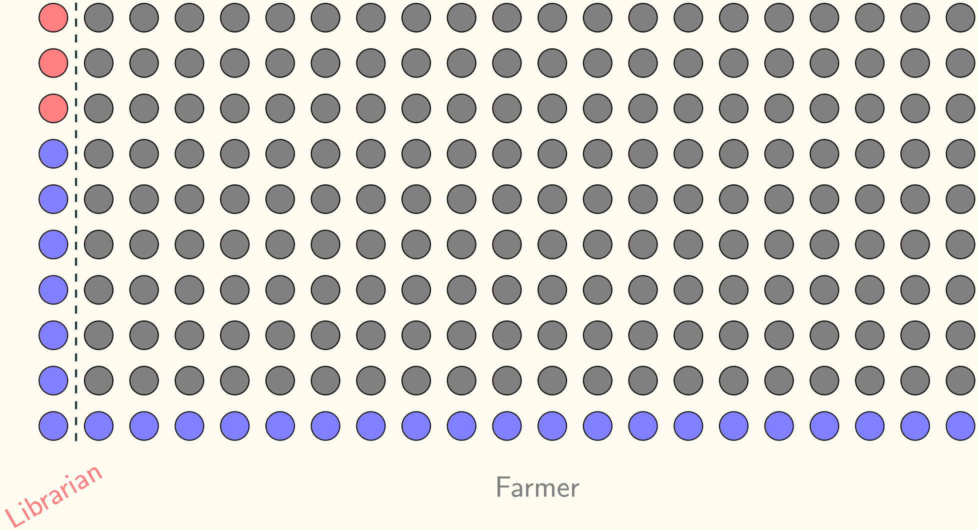
Abstract

This article described three heuristics that are employed in making judgements under uncertainty: (i) representativeness, which is usually employed when people are asked to judge the probability that an object or event A belongs to class or process B; (ii) availability of instances or scenarios, which is often employed when people are asked to assess the frequency of a class or the plausibility of a particular development; and (iii) adjustment from an anchor, which is usually employed in numerical prediction when a relevant value is available. These heuristics are highly economical and usually effective, but they lead to systematic and predictable errors. A better understanding of these heuristics and of the biases to which they lead could improve judgements and decisions in situations of uncertainty.

Amos Tversky & Daniel Kahneman

"Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail. How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, or physician)?"

Who Is Steve



When To Use The Bayes' Theorem

You have a hypothesis	You have observed some evidence	You want
The person carries the virus; Steve is a librarian	Test result is positive; Steve's characters	Probability of the hypothesis given the evidence, $\mathbb{P}(H\|E)$

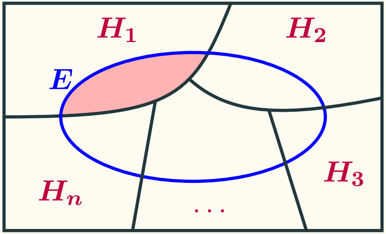
The Alternative Form Of The Bayes' Theorem

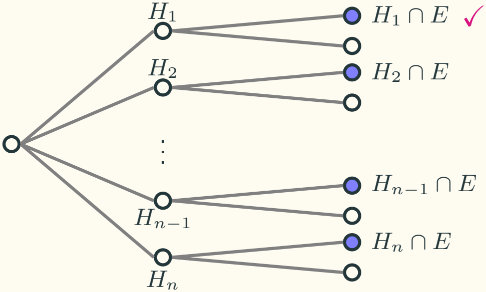
Bayes' Theorem

$$\mathbb{P}(H_i|E) = \frac{\mathbb{P}(H_i)\cdot\mathbb{P}(E|H_i)}{\mathbb{P}(E)}$$

$$= \frac{\mathbb{P}(H_i)\cdot\mathbb{P}(E|H_i)}{\sum_{i=1}^{n}\mathbb{P}(H_i)\cdot\mathbb{P}(E|H_i)}$$

Sample space Ω





The Bayes' Theorem

$$\mathbb{P}\left(H_i|E\right) = \frac{\mathbb{P}\left(E|H_i\right)}{\sum_{i=1}^{n} \mathbb{P}\left(H_i\right) \cdot \mathbb{P}\left(E|H_i\right)} \cdot \mathbb{P}\left(H_i\right)$$

$\mathbb{P}\left(H_i\right)$: **prior probability**

$\mathbb{P}\left(H_i|E\right)$: **posterior probability**

The Bayesian Search

- The 4th H-bomb from American B-52 (1966)
- Air France 447 (2009 - 2011)
- Malaysian Air Flight 370 (2014 -)
- **USS Scorpion (SSN-589)** (1968)

US Navy photo #NH_97214 & 1136658

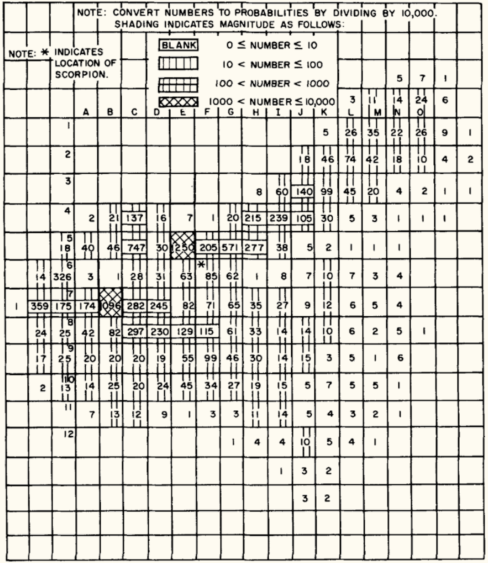
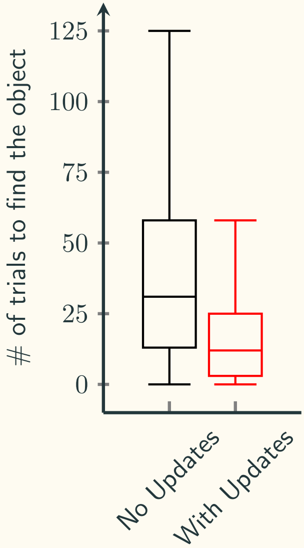


FIGURE 2. Overall A Priori distribution for *Scorpion* search

Richardson & Stone - *Operations analysis during the underwater search for Scorpions* (1971)

Simulation of The Bayesian Search

0.14	0.07	0.11
0.22	0.00	0.03
0.17	0.21	0.04



One Simulation Result

0.14	0.07	0.11
0.22	0.00	0.03
0.17	0.21	0.04

→

0.15	0.07	0.11
0.19	0.00	0.03
0.18	0.22	0.04

→

0.15	0.07	0.12
0.19	0.00	0.03
0.187	0.188	0.04

→

0.15	0.08	0.12
0.16	0.00	0.03
0.194	0.195	0.04

→

0.16	0.08	0.12
0.16	0.00	0.03
0.20	0.16	0.04

→

0.16	0.08	0.12
0.169	0.00	0.03
0.168	0.167	0.04